THE EIGENVALUE MYTH AND THE DIMENSION-REDUCTION FALLACY*

Marion F. Shaycoft
American Institutes for Research
Palo Alto, California

One definition of myth is "a person or thing existing only in imagination". The expression "eigenvalue myth" isn't intended to imply that eigenvalues themselves exist only in imagination, but rather that one of their attributes does -- namely the mysterious power that the eigenvalues are widely believed to possess, of indicating which factors (or more properly, which components) are "statistically significant" or "important" or "meaningful" or "reliable". Kaiser, who popularized the idea of retaining in a component analysis only those components with eigenvalues greater than 1, surely never claimed that those were the only components that were statistically significant; in proposing the principle he presented it merely as a handy and generally useful rule of thumb -- certainly not as an infallible rule. Let me quote him, abridging slightly:

> "I chose [italics mine] to accept only those principal components as being useful which were able to make a larger variance contribution than an individual test.... Now from a factor analytic viewpoint, where by factor analysis I mean the study of the structural inter-relationships among variables, such a rule of behavior is logically dubious. But... I seem to have stumbled onto what appears to be an excellent rule of behavior for the number of factors which may adequately be determined under the strict factor analytic model.... I don't deny that prior information for a given study might... lead to a better answer for this particular study." (Kaiser, 1960, 1).

Nothing in that statement suggests that the eigenvalue-greater-than-1 principle is a sacred law. A statement Kaiser makes elsewhere,

---

* A slightly abridged version of this paper was presented at the Annual Meeting of the American Educational Research Association, in Minneapolis, 6 March 1970.

however, is probably partly responsible for the widespread belief that the principle has mathematical support. He says:

> "...for a principal component to have positive Kuder-Richardson reliability, it is necessary and sufficient that the associated eigenvalue be greater than one..." (Kaiser, 1960, 2)

I have been unable to track down his formulas supporting this statement or anything further on this point and consequently am not certain exactly what the expression "Kuder-Richardson reliability of a component" refers to. Perhaps the reference is to a generalized KR20, with the original variables, appropriately weighted, playing the role that test items play in the original Kuder-Richardson formulation. If this supposition or any alternate supposition I can think of is correct it seems to me that KR is an utterly inappropriate way of determining the reliability of a component. The assumptions underlying KR20 (even in the generalized version) are numerous, restrictive, and very definitely unmet in the case of principal component scores. As a matter of fact, it can be proved quite readily that <u>any</u> linear composite of variables that have some reliability will itself necessarily have some reliability -- and that statement holds even if the linear composite happens to be an estimation of a principal component score, for a component with an eigenvalue less than 1.

One of the things I propose to do in this paper is to disprove, by demonstration, that an eigenvalue greater than one necessarily corresponds to a component that is "reliable" in any meaningful sense. Evidence will also be presented that I regard as <u>suggesting strongly</u> that some components with eigenvalues below one can be meaningful and important,*

---

* Humphreys (1964) has presented empirical evidence of a substantive nature that particularly when N is large the eigenvalue rule may give far too few factors.

which in turn implies that they must have some useful degree of reliability. (Note that I don't claim to prove that components with eigenvalues below one are important. "Importance" is a value judgment, and therefore not subject to proof.)

The data analysis that provides the evidence on all this starts with a 14-variable correlation matrix based on 3689 twelfth-grade boys in Project TALENT. Ten of the 14 variables are test scores (on selected tests from the TALENT battery). The remaining four variables are computer-generated random numbers.* The correlation matrix is in Table 1. A complete principal components analysis was carried out. The results are shown in Table 2. The interesting thing about them is the way the four random variables are accounted for by components P2 - P5. All four of these components have eigenvalues close to 1, with two of them slightly above 1 and two slightly below. Suppose we take the eigenvalue-greater-than-1 rule literally, and therefore rotate just the first three components. Table 3 shows the results, which are almost identical for varimax and quartimax rotations. I don't think that anyone unfamiliar with the 14 original variables would suspect, just on the basis of looking at the rotated components in Table 3, or the principal components in Table 2, that four of the variables were random numbers. Nor would further rotation "by hand" clarify the matter.

Knowing what we know about the nature of variables 11-14, I think all of us would agree that the second and third rotated components have essentially zero reliability and are quite meaningless. The same thing is true, of course, of the four principal components P2 through P5. P1 is obviously a general component, bearing a large share of the meaningful variance. But what about the last nine components,

---

* Other researchers who have utilized random numbers in an effort to solve the number-of-factors problem through methodological studies are Horn (1965), Linn (1968), and Humphreys and Ilgen (1969). Their approaches were quite different from that of the study reported here.

TABLE 1. Correlation matrix

First 10 variables: TALENT test scores
Last 4 variables: Random numbers

Cases: A representative 10% sample of those 12th-grade boys in Project TALENT having scores on all 10 of the test variables.

No. of cases: N=3689 12th-grade boys

	TALENT test		R-102 1	R-103 2	R-106 3	R-107 4	R-112 5	R-230 6	R-250 7	R-282 8	R-290 9	R-312 10	X1 11	X2 12	X3 13	X4 14	$\overline{X}$	s
1	R-102	Vocabulary		.726	.690	.733	.568	.614	.723	.417	.506	.615	-.014	.012	-.022	-.003	14.08	3.80
2	R-103	Literature Info			.648	.672	.398	.582	.700	.321	.442	.575	-.020	.012	-.024	.020	14.12	4.74
3	R-106	Math Information				.764	.405	.608	.639	.456	.554	.840	-.026	.024	-.021	.004	11.42	6.18
4	R-107	Physical Science Information					.515	.558	.650	.442	.521	.689	-.008	.022	-.026	-.005	10.44	4.37
5	R-112	Mechanical Information						.345	.441	.400	.341	.349	-.006	.010	-.004	-.003	13.37	3.36
6	R-230	English							.659	.366	.501	.653	-.029	-.003	-.021	.005	82.77	13.34
7	R-250	Reading Comprehension								.440	.577	.616	-.029	.005	-.023	.020	32.99	10.65
8	R-282	Visualization in Three Dimensions									.579	.442	-.026	.015	-.009	-.001	9.75	3.45
9	R-290	Abstract Reasoning										.558	-.027	.018	.004	.002	9.61	3.00
10	R-312	Math II. Introd. H.S. Math											-.029	.029	-.011	-.006	12.41	5.61
11	X-1	Random numbers*												.001	-.009	-.028	.4922	.2885
12	X-2	Random numbers*													.019	-.005	.4997	.2891
13	X-3	Random numbers*														.010	.5013	.2886
14	X-4	Random numbers*															.5000	.2880

* Four-digit random numbers between .0000 and .9999, with an approximately rectangular distribution.

TABLE 2: Principal components loadings

		P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11	P12	P13	P14	$\sum_1^3 a^2$	$\sum_1^{14} a^2$
1. R-102	Vocab.	.859	-.038	-.020	.011	.038	.133	.238	.079	.088	.012	.017	.149	.383	-.044	.739	1
2. R-103	Lit. Info.	.795	-.016	-.058	-.014	.074	.306	.101	.185	.301	.118	.202	-.259	-.088	-.028	.635	1
3. R-106	Math Info.	.865	-.004	.004	-.023	.008	.118	-.167	-.345	.023	-.008	-.050	-.083	.056	.279	.748	1
4. R-107	Phys. Sci. Info.	.854	-.039	.006	-.006	.029	.091	.101	-.244	.156	-.041	.092	.324	-.221	-.048	.731	1
5. R-112	Mechanical Info.	.594	-.034	.041	.060	.003	-.251	.690	-.090	-.241	-.119	-.008	-.132	-.063	.008	.356	1
6. R-230	English	.766	.004	-.051	.001	.002	.174	-.190	.260	-.471	.126	.168	.082	-.043	.045	.589	1
7. R-250	Rdg. Comp.	.839	.009	-.043	.005	.025	.089	.007	.332	.061	.005	-.397	.030	-.107	.019	.706	1
8. R-282	Vis. 3 Dimensions	.604	.048	.087	.037	-.112	-.671	-.092	-.006	.088	.380	.013	.006	.008	-.006	.375	1
9. R-290	Abst. Reas.	.711	.057	.064	.033	-.079	-.390	-.287	.192	.082	-.434	.109	-.024	.016	.004	.513	1
10. R-312	Introd. H.S. Math	.830	.007	.019	-.023	-.015	.106	-.290	-.321	-.153	-.015	-.104	-.150	.032	-.233	.689	1
11. X-1		-.033	-.644	.229	.326	.641	-.083	-.090	.012	-.008	.004	-.003	-.006	.000	.001	.468	1
12. X-2		.022	.137	.731	-.633	.208	.030	.026	.037	-.012	.005	.000	.003	-.000	.002	.554	1
13. X-3		-.025	.459	.544	.681	-.074	.152	.008	.001	.006	.015	-.001	.005	-.002	.002	.507	1
14. X-4		.005	.618	-.340	-.028	.698	-.113	-.006	-.030	-.011	-.004	.004	.007	.005	-.003	.498	1
Eigenvalue (= Σa^2)		6.052	1.036	1.020	.979	.974	.892	.801	.547	.445	.379	.261	.249	.225	.140	8.108	14
% of variance (=% of trace)		43.23	7.40	7.28	6.99	6.96	6.37	5.72	3.91	3.18	2.71	1.86	1.78	1.61	1.00	57.91	100

$\sum_1^3 a^2$ = sum of squares for first three principal components.

$\sum_1^{14} a^2$ = sum of squares for all 14 principal components.

TABLE 3: Varimax and Quartimax Factor Loadings for rotation of first 3 principal components

		VARIMAX LOADINGS V1	V2	V3	Σb^2	QUARTIMAX LOADINGS Q1	Q2	Q3	Σb^2
1	R-102	.859	-.015	-.039	.740	.858	-.019	-.042	.738
2	R-103	.794	.020	-.065	.635	.794	.016	-.067	.635
3	R-106	.865	.006	-.004	.748	.865	.002	-.006	.748
4	R-107	.854	-.028	-.016	.730	.854	-.032	-.019	.731
5	R-112	.595	-.041	.019	.356	.595	-.044	.017	.356
6	R-230	.766	.034	-.050	.590	.766	.031	-.052	.590
7	R-250	.839	.037	-.041	.707	.839	.033	-.043	.707
8	R-282	.605	.015	.095	.375	.605	.013	.093	.375
9	R-290	.711	.034	.077	.513	.712	.030	.075	.513
10	R-312	.830	.009	.015	.689	.830	.005	.012	.689
11	X-1	-.025	-.681	-.060	.468	-.028	-.681	-.060	.468
12	X-2	.029	-.180	.721	.553	.030	-.180	.721	.553
13	X-3	-.023	.190	.686	.507	-.020	.190	.687	.508
14	X-4	-.004	.704	-.051	.498	-.001	.704	-.052	.498
	Σb^2	6.053	1.035	1.022		6.053	1.035	1.024	

P6 through P14? Are we to relegate them to limbo because their eigenvalues are below 1? I don't think we should.

This particular set of 14 variables was designed and originally used for a purpose* entirely unrelated to the topic of the present paper and therefore is not especially well suited to factor analysis. Nevertheless rotation of the principal components does yield meaningful factors beyond the obvious general one suggested by the first principal component. The results of varimax and quartimax rotations of all 14 principal components are shown in Table 4. (Both kinds of rotations were carried out in order to see which would provide a better starting point for subsequent non-blind rotations.)

* The 14-variable matrix was obtained and used by Shaycoft in a paper entitled "Converting an Inconsistent Matrix into a Consistent Non-Singular Matrix," presented at the 1968 AERA Convention.

For both the varimax and the quartimax, the components shown in the last four columns account for virtually all the random variable variance. It would be obvious to anyone inspecting Table 4, as it also would to anyone inspecting the initial correlation matrix, that all the variance of the last four variables (X1 - X4) is unique. What is not obvious from inspection is that none of this unique variance is specific variance; it is all unreliability.

A comparison of Tables 3 and 4 makes it clear that the stage at which components are eliminated from further consideration has an important effect on results. The second and third varimax or quartimax components in Table 3 (and the first varimax as well) are not even remotely like any of the varimax (or quartimax) components in Table 4. And certainly the Table 4 versions of the components corresponding to the random variables are far superior to the Table 3 versions. Furthermore, as we shall see later, the Table 4 components corresponding to the test variables provide useful information beyond that provided by the single such component in Table 3. It appears, then, that even if one is going to retain only three components after rotation (the number indicated by the eigenvalue-greater-than-one rule), it is unsafe to cut the number down to three <u>before</u> rotation.

Subsequent rotations, starting from the quartimax results, produced the final pattern, shown in its entirety in Table 5. The results would probably have been somewhat more clear-cut in terms of interpretability if they had been based on factor analysis instead of component analysis, but the latter procedure was of course dictated by the nature of this methodological study. Despite the not quite suitable set of variables and the not quite suitable methodology, Table 5 shows three very solid common factors which not only seem sound intuitively but which also closely resemble factors that showed up in a much larger factor analysis study involving 95 TALENT variables (including the ten in the present study) and two separate factor analyses, one for boys and one for girls (Shaycoft, 1967, Chapter 6).

TABLE 4: Varimax and Quartimax Factor Loadings for rotation of 14 principal components

V A R I M A X   L O A D I N G S

		V1	V2	V3	V4	V5	V6	V7	V8	V9	V10	V11	V12	V13	V14
1.	R-102	.351	.235	.137	.686	.226	.308	.160	.199	.347	.020	-.002	.003	-.010	-.004
2.	R-103	.315	.209	.083	.169	.203	.154	.128	.157	.845	.015	-.007	.004	-.012	.013
3.	R-106	.789	.144	.167	.160	.170	.149	.177	.200	.250	.343	-.010	.010	-.011	.003
4.	R-107	.468	.169	.158	.185	.160	.254	.168	.702	.284	.021	.003	.011	-.014	-.006
5.	R-112	.149	.100	.158	.119	.091	.940	.093	.110	.122	.009	-.000	.003	.000	-.002
6.	R-230	.347	.186	.116	.125	.848	.119	.170	.098	.207	.014	-.013	-.007	-.010	.002
7.	R-250	.316	.752	.162	.178	.269	.183	.227	.141	.313	.014	-.013	-.002	-.012	.014
8.	R-282	.200	.096	.919	.067	.096	.169	.228	.080	.075	.010	-.012	.006	-.004	-.001
9.	R-290	.275	.152	.275	.089	.162	.116	.867	.098	.128	.012	-.012	.008	.007	.001
10.	R-312	.876	.141	.156	.090	.244	.106	.189	.105	.162	-.188	-.013	.016	-.002	-.006
11.	X-1	-.011	-.006	-.009	-.001	-.009	-.000	-.008	.001	-.005	-.001	1.000	.001	-.005	-.014
12.	X-2	.013	-.000	.005	.002	-.004	.003	.005	.004	.003	.001	.001	1.000	.010	-.003
13.	X-3	-.006	-.006	-.003	-.004	-.006	-.000	.004	-.006	-.008	-.001	-.005	.010	1.000	.005
14.	X-4	-.002	.006	-.001	-.001	.002	-.002	.001	-.002	.008	.000	-.014	-.002	.005	1.000
Σb^2		2.190	.811	1.088	.641	1.042	1.190	1.030	.666	1.182	.155	1.001	1.001	1.001	1.001
% of variance		15.6	5.8	7.8	4.6	7.4	8.5	7.4	4.8	8.4	1.1	7.2	7.2	7.2	7.2

Q U A R T I M A X   L O A D I N G S

		Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	Q14
1.	R-102	.825	.055	.031	.517	.020	.200	.010	-.005	.075	.048	.002	-.001	-.006	-.005
2.	R-103	.780	.066	-.023	.044	.022	.046	-.011	-.001	.617	.027	-.004	-.000	-.008	.015
3.	R-106	.941	-.146	.057	-.150	-.078	-.008	.006	-.145	-.114	.165	-.007	.009	-.005	.001
4.	R-107	.861	-.039	.053	-.005	-.067	.133	.009	.478	-.002	.051	.008	.008	-.009	-.007
5.	R-112	.445	.022	.112	.034	.008	.887	.027	.019	.015	.003	.002	.001	.003	-.003
6.	R-230	.711	.053	.036	.009	.697	.018	.061	-.021	.015	-.006	-.012	-.012	-.006	.001
7.	R-250	.781	.595	.070	.037	.086	.075	.098	-.020	.074	.024	-.011	-.007	-.008	.015
8.	R-282	.437	.021	.879	.006	.017	.115	.150	.008	-.008	-.004	-.010	.005	-.001	-.002
9.	R-290	.581	.043	.230	.003	.043	.040	.777	.002	-.005	-.005	-.010	.006	.011	.000
10.	R-312	.885	-.102	.061	-.144	.035	-.045	.042	-.113	-.143	-.380	-.011	.015	.004	-.009
11.	X-1	-.020	-.002	-.007	.001	-.004	.002	-.005	.001	-.001	.001	1.000	.001	-.005	-.014
12.	X-2	.015	-.001	.003	-.000	-.004	.001	.003	.001	.000	-.001	.001	1.000	.010	-.003
13.	X-3	-.019	-.002	-.001	-.001	-.002	.002	.005	-.001	-.002	-.000	-.005	.010	1.000	.005
14.	X-4	.003	.003	-.001	-.001	.001	-.002	.000	-.000	.003	.000	-.014	-.003	.005	1.000
Σb^2		5.542	.400	.856	.315	.508	.869	.642	.264	.426	.178	1.001	1.001	1.001	1.001
% of variance		39.6	2.9	6.1	2.2	3.6	6.2	4.6	1.9	3.0	1.3	7.1	7.1	7.1	7.1

TABLE 5: Final factor pattern (orthogonal)

			FACTOR LOADINGS														Communality corresp. to first three factors
			General factor for test variables	Important common factors		Factors accounting mostly for unique variance (reliable and unreliable) in the test variables						Factors accounting for the random variables					
										Bipolar factors							
			F1	F2	F3	F4	F5*	F6	F7	F8*	F9	F10*	F11*	F12*	F13*	F14*	h^2
			Gen. verbal	Math	Spatial	Vocab. acquis.	Engl.	Mech. Info.	Abst. Reas.	Phys. SciInf.	R103 -R250	R106 -R312					
1	R-102	Vocab	.771	.008	.033	.634	.020	-.017	.004	-.005	.020	.048	.002	-.001	-.006	-.005	.596
2	R-103	Lit. Info.	.901	-.050	-.025	.036	.022	.036	-.006	-.001	.424	.027	-.004	-.000	-.008	.015	.815
3	R-106	Math "	.744	.599	.057	.160	-.078	-.067	-.004	-.145	.006	.165	-.007	.009	-.005	.001	.916
4	R-107	Phys.Sci."	.746	.377	.054	.244	-.067	.052	-.001	.478	.022	.051	.008	.008	-.009	-.007	.702
5	R-112	Mech. "	.405	.144	.115	.418	.008	.792	.007	.019	-.004	.003	.002	.001	.003	-.003	.198
6	R-230	English	.651	.246	.046	.149	.697	-.035	.054	-.021	-.024	-.006	-.012	-.012	-.006	.001	.486
7	R-250	Rdg.Comp.	.924	-.083	.086	.021	.086	.073	.084	-.020	-.332	.024	-.011	-.007	-.008	.015	.868
8	R-282	Vis. 3 D	.391	.167	.891	.136	.017	.073	-.005	.008	-.020	-.004	-.010	.005	-.001	-.002	.975
9	R-290	Abst.Reas.	.526	.213	.361	.132	.043	-.006	.726	.002	-.033	-.005	-.010	.006	.011	.000	.452
10	R-312	Math II	.701	.562	.068	.134	.035	-.096	.031	-.113	-.044	-.380	-.011	.015	.004	-.009	.812
11	Random variables	X-1	-.018	-.007	-.008	-.002	-.004	.003	-.004	.001	.001	.001	1.000	.001	-.005	-.014	.000
12		X-2	.013	.007	.004	.004	-.004	-.000	.002	.001	.001	-.001	.001	1.000	.010	-.003	.000
13		X-3	-.018	-.005	.000	-.003	-.002	.003	.005	-.001	-.000	-.000	-.005	.010	1.000	.005	.000
14		X-4	.005	-.002	-.001	-.002	.001	-.001	.000	-.000	.000	.000	-.014	-.003	.005	1.000	.000
	Σb^2		4.880	.981	.960	.740	.508	.657	.538	.264	.295	.178	1.001	1.001	1.001	1.001	6.820
	% of variance		34.84	7.00	6.85	5.28	3.63	4.69	3.84	1.89	2.11	1.27	7.15	7.15	7.15	7.15	48.69

* Same as corresponding quartimax factor

The three confirmed factors are general verbal, math, and spatial. All three are somewhat contaminated by unique variance in the present study, but that is probably inevitable in a component analysis, which merges unique variance with common variance inextricably. The important point about factors F_2 and F_3 is that they are meaningful and important factors that wouldn't have appeared in this analysis if principal components with eigenvalues less than 1 hadn't been rotated.* This is evidence that eigenvalues don't tell anything about the reliability of a component.

Suppose instead of putting 1's in the diagonal of the original correlation matrix we put reliability coefficients in -- or better yet, some reasonable estimates of communality. Wouldn't this solve the problem of the random variables? Of course it would. If the entries in the row and column corresponding to any one variable were all close enough to 0 to convince the researcher that the variable was in effect a random one, or one with nothing but unique variance, presumably he would have enough sense to throw it out of his analysis. But this form of dimension reduction would be a decision by a person, not by an eigenvalue. Even with something other than unities in the diagonal, eigenvalues don't tell us about the reliability of a factor.

But there is another possibility still to be considered. Could it be that since measurement errors don't have any direct bearing on eigenvalues, sampling errors might? (The term "sampling errors" is used here in its usual sense, referring to a sample of persons, not

---

* As for the remaining 11 factors, factor F4 in Table 5 looks a little bit like a genuine common factor, but it is probably not, because no factor looking much like it appeared in the large factor analysis done earlier (Shaycoft, 1967). Rather, it is probably just some kind of jumble of unique variance. The next six factors, F5-F10, primarily represent unique variance. The last two of these (F9 and F10) seem intractably bipolar (unless one is willing to do without meaningful common factors such as F2) and therefore probably represent unreliability components responsible for discrepancies between two rather closely related variables. The last 4 factors (F11-F14) are the quartimax factors accounting for the four random variables.

to a sample of variables.) When there are eigenvalues that are small but non-zero, there seems to be a tendency among some to attribute the departure from 0 to sampling errors. To know whether this tendency is a defensible one, it is necessary to understand precisely what is implied by the concept of sampling error, when it is applied to an eigenvalue. Dimension reduction occurs when an eigenvalue is exactly zero, which in turn occurs only if one variable is a linear function of some or all the others. This implies either a zero-order or multiple correlation of 1 somewhere in the set of variables. If an eigenvalue's departure from zero is attributable to sampling it means that the population value of the eigenvalue is zero, but that the sample on which the statistics are based is such that for that sample the eigenvalue is _not_ exactly zero. There is a correlation whose population value is 1, so that everybody in the population lies exactly on the regression line (or the regression plane or hyperplane in the case of a multiple correlation coefficient). But as soon as you take just a subset of the population, leaving their scores on all variables intact, because -- remember -- we are talking now about sampling errors not measurement errors, some of the people in the selected subset somehow are no longer on the regression line, or plane, or hyperplane.

But if the _population_ value of a correlation coefficient is 1, for absolutely no _sample_ of that population can the correlation be less than 1. Suppose, for instance, that Variable X is a person's height expressed in feet and fractions of a foot; and that Variable Y is that same height, expressed in inches and fractions of an inch. The population value of the correlation between X and Y is 1, of course. Now suppose we have a sample from that population. It amuses me to fantasize about someone in the sample explaining that though he is 6.25 feet tall, he is still only 67 inches tall -- and that it is all due to sampling errors, which caused the poor fellow to fall off the regression line! If that seems as absurd to the reader as it does to the writer, it will be clear why sampling errors cannot possibly reduce a perfect correlation and thus cannot possibly raise a zero eigenvalue above zero.

The term "statistical significance" is sometimes used with reference to a principal component. Presumably the statement that a principal component is statistically significant means that its eigenvalue is significantly greater than zero. But for a principal component to exist at all, it must have an eigenvalue greater than zero, and I believe I have shown that if the sample eigenvalue is greater than zero the population eigenvalue must also be -- so that if a principal component exists at all in a sample it also exists in the population. This leads to the conclusion that the entire concept of "statistical significance of a principal component" is essentially meaningless.

Neither sampling errors nor measurement errors can create new dimensions which didn't already exist in the population. However these errors can and do have both qualitative and quantitative effects on the relationship between the components and the original variables from which they were derived. Furthermore it is quite possible, as a result of sampling errors, for dimensions that exist in the population not to exist in a sample. But eigenvalues won't give any definite indication of this aberration, either.

If eigenvalues tell us little or nothing about the "reliability" and "statistical significance" of a component or factor, do they at least tell us something about its "importance"? Again, no. Even a very tiny eigenvalue may correspond to a principal component that is worth rotating. And even a factor that accounts for only a very small proportion of the total common variance still may be meaningful, interpretable, and "important" for some specific purpose. A common factor that accounts for a substantial proportion of certain variables' variance may still represent only a very small proportion of the total variance of all variables combined, if there are a great many variables and a great many common factors in the analysis.*

---

* For instance the 95-variable factor analysis on TALENT data, referred to earlier, (Shaycoft, 1967) resulted in 40 meaningful common factors, accounting for about two-thirds of the common variance. Some of these factors accounted for only a fraction of a percent of the total variance. But that was because there were so many factors, which in turn was due to the very large number of variables.

Therefore if one decides to throw away some dimensions in component analysis, the decision must be based on one's knowledge of the number and nature of the variables in the analysis, understanding of the goals of the study, statistical insight, and common sense. In other words the researcher himself must take responsibility for the decision rather than expecting the eigenvalues to decide.

Perhaps it is fortunate that the eigenvalue rule doesn't work -- because use of the rule in its original form implies that one should be doing a component analysis rather than factor analysis; in other words it suggests 1's in the diagonal of the correlation matrix. As has already been implied, confusions and ambiguities can result from this practice, which is a particularly unsuitable one when the primary purpose of one's analysis is to gain an understanding of the factors underlying the variables and accounting for their intercorrelations, rather than to obtain scores on the artificial components into which the original variables can be split. (Putting communalities in the diagonal in no way denies the importance of reliable specific variance, which in some contexts can be even more important than common variance. But the best and most direct way to get a reasonable estimate of the magnitude of specific reliable variance is to obtain a good estimate of the variable's communality somehow or other, and to subtract it from a good estimate of the variable's reliability.)

Just as the existence of a component in a sample proves its existence in the population, a common factor that exists in a sample must also exist in the population (regardless of what the eigenvalues look like).

Furthermore, a common factor must have perfect reliability -- although, as in the case of component analysis, sampling error may somewhat obscure its character. Factors are of course nothing more than hypothetical constructs. The term "common factors", as used here, refers to the hypothetical constructs themselves, not to any estimates of an individual's scores on them. That is a very different matter, and it bears on the topic under consideration, since the intention of computing factor scores is often advanced as an argument for preferring component analysis to factor analysis and also as a practical reason for the drastic dimension reduction that the eigenvalue rule results in, since the fewer the dimensions remaining, the fewer factor scores need be computed.

These arguments are weak, however, since the practical advantages of routinely computing factor scores for individuals are more apparent than real. Use of component scores or factor scores doesn't reduce the number of variables on which scores are required for each individual; in fact in one sense it increases it, since computation of factor or component scores requires knowledge of scores on all the original variables. Furthermore using factor or component score estimates as a replacement for the original test scores or other original data takes us one step further from reality and adds brand-new elements of uncertainty and ambiguity. Quite apart from the sampling errors and measurement errors that presumably affect the original data are the errors of estimation that are inevitable when one is estimating scores on unobservable hypothetical constructs such as factors -- however meaningful they may be. And as Harris's study of factor scores (Harris, 1967) has shown in effect, the more effort one makes to get meaningful common factors (for instance through such devices as using communality estimates (to keep unique variance out of the "common factors") and rotating the initial solution (to improve the interpretability of the factors), the less possible it becomes to get reasonable factor score estimates. More specifically, with an orthogonal rotated factor solution, factor score estimates may be univocal or they may be uncorrelated but they cannot be both -- and both would be desirable. At the risk of oversimplifying considerably, all this can be summarized by saying that one can have <u>bad</u> estimates of <u>good</u> factors, or <u>good</u> estimates of <u>bad</u> factors, or even <u>bad</u> estimates of <u>bad</u> factors -- but that <u>good</u> estimates of <u>good</u> factors seem to be beyond present methodology.

In discussing various kinds of errors (errors of measurement, sampling errors, etc.) I have not discussed the concept of so-called "errors" resulting from the sampling of test variables, and I do not intend to discuss it, except to mention that in my opinion the entire concept of random sampling from a hypothetical infinite population of tests was so thoroughly and so effectively demolished by Loevinger a number of years ago (Loevinger, 1965) that I marvel at its continued use by psychologists.

In summary, then, eigenvalues tell one nothing about the reliability of a component. Nor do they tell us about its "statistical significance" or its "importance." Factor analysts who use the eigenvalue rule should bear this in mind and not follow it blindly. Rules of thumb undoubtedly have a place in the scheme of things -- but when it comes to thinking, thumbs are no substitute for heads.

REFERENCES

Harris, C. W. On factors and factor scores. Psychometrika, 1967, 32, 363-379.

Horn, J. L. A rationale and test for the number of factors in factor analysis. Psychometrika, 1965, 30, 179-185.

Humphreys, L. G. Number of cases and number of factors: an example where N is very large. Educational and Psychological Measurement, 1964, 24, 457-466.

Humphreys, L. G. & Ilgen, D. R. Note on a criterion for the number of common factors. Educational and Psychological Measurement, 1969, 29, 571-578.

Kaiser, H. F. Comments on communalities and the number of factors. Paper read at a conference, The Communality Problem in Factor Analysis, Washington University, St. Louis, 1960, under the sponsorship of Philip H. Du Bois.

Kaiser, H. F. The application of electronic computers to factor analysis. Educational and Psychological Measurement, 1960, 20, 141-151.

Linn, R. L. A Monte Carlo approach to the number of factors problem. Psychometrika, 1968, 33, 37-71.

Loevinger, Jane. Person and population as psychometric concepts. Psychol. Rev., 1965, 72, 143-155.

Shaycoft, Marion F. The high school years: growth in cognitive skills. (Cooperative Research Project No. 3051) Project TALENT Office, 1967.

SELECTED REFERENCES ON PROJECT TALENT

Flanagan, J. C., Dailey, J. T., Shaycoft, Marion F., Gorham, W. A., Orr, D. B., & Goldberg, I. Design for a study of American Youth, 248 pp. Boston: Houghton Mifflin, 1962.

Flanagan, J. C., Dailey, J. T., Shaycoft, Marion F., Orr, D. B., & Goldberg, I. Studies of the American High School. (Cooperative Research Project No. 226) Project TALENT Office, 1962. 375 pp.

Flanagan, J. C., Davis, F. B., Dailey, J. T., Shaycoft, Marion F., Orr, D. B., Goldberg, I., & Neyman, C. A., Jr. The American High School Student. (Cooperative Research Project No. 635) Project TALENT Office, 1964. 738 pp.

Flanagan, J. C., Cooley, W. W., Lohnes, P. R., Schoenfeldt, L. F., Holdeman, R. W., Combs, Janet, & Becker, Susan. Project TALENT One-Year Follow-Up Studies. (Cooperative Research Project No. 2333) Project TALENT Office, 1966. 348 pp.

Shaycoft, Marion F., Dailey, J. T., Orr, D. B., Neyman, C. A., Jr., & Sherman, S. E. Studies of a Complete Age Group--Age 15. (Cooperative Research Project No. 566) Project TALENT Office, 1963. 288 pp.

Shaycoft, Marion F. The High School Years: Growth in Cognitive Skills. (Cooperative Research Project No. 3051) Project TALENT Office, 1967. 376 pp.